

Dynamic Bipartite Stochastic Blockmodel Regression for Network Data

Application to State and Intergovernmental Organization Networks

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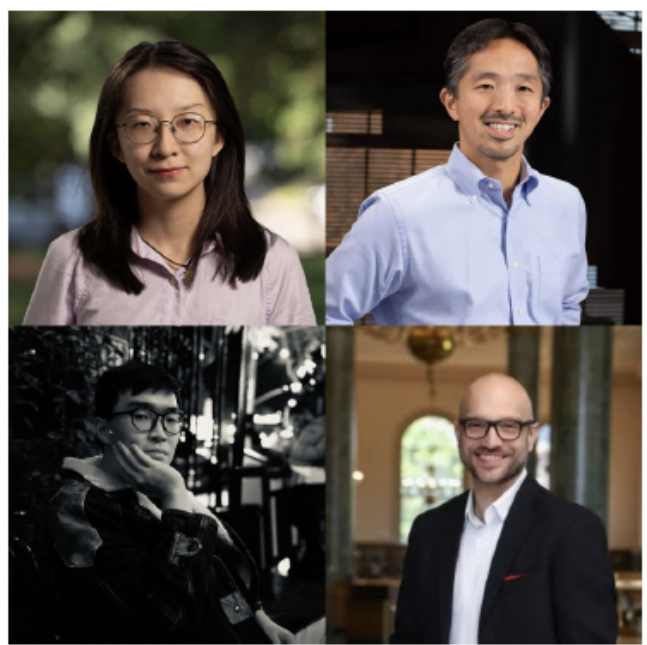
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Motivation

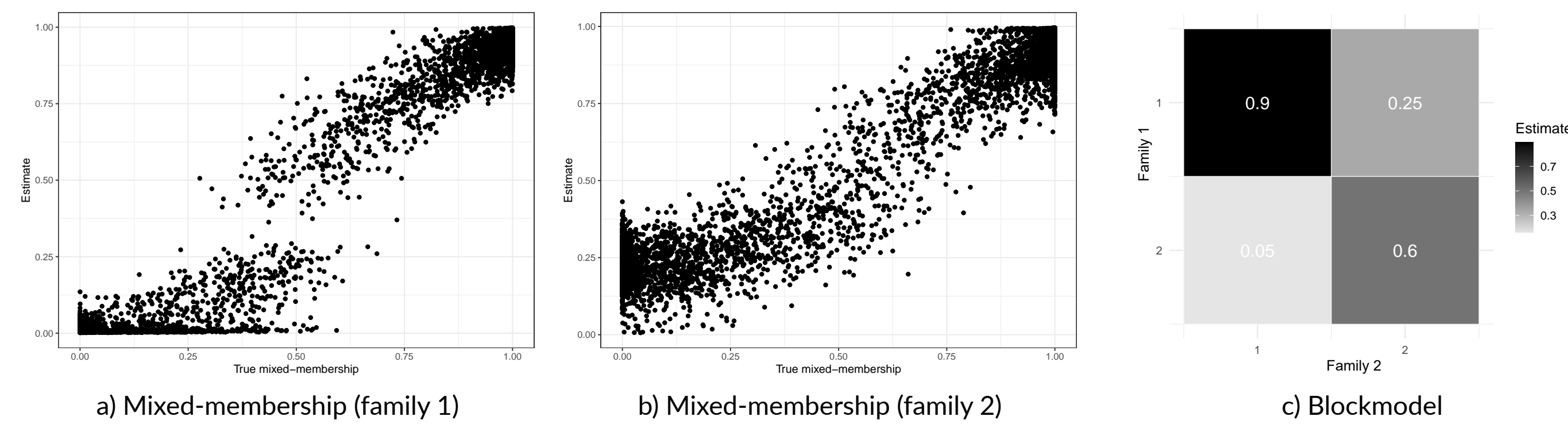
- Most political networks are:**
 - Bipartite** — two node types, ties only across types (e.g., *states-treaties*, *legislatures-bills*, *lobbyists-politicians*)
 - Dynamic** — ties evolve over time
- Problem:** Use static/ projected unipartite models \rightsquigarrow bias and spurious clustering
- Our contribution:** **Dynamic Bipartite MMSBM**

	Dynamic: No	Dynamic: Yes
Bipartite: No	Box-Steffensmeier et al. (2019); Siegel & Badaan (2020); Weschle (2018); Naidu et al. (2021); Cruz et al. (2020); Weschle (2019); Bouchier & Thies (2019); Bodea & Hicks (2015); Jiang & Zeng (2020); Shaffer (2022); Franzese et al. (2012); Lo et al. (2023); Goddard (2018); Edgerton (2024); Battaglini et al. (2020); Nyhan & Montgomery (2015); Box-Steffensmeier et al. (2020); Oklobdzija (2024); Aaroe & Peterson (2020); Kim et al. (2019); Blair et al. (2022); Cho & Fowler (2010); Crammer & Desmarais (2011); Fishman and Davis (2022); Abi-Hassan et al. (2023)	Kim et al. (2020); Gilardi et al. (2020); Nieman et al. (2021); Harden et al. (2023); Uppala and Desmarais (2023)
Bipartite: Yes	Sweet (2021); Kim & Kunisky (2020)	

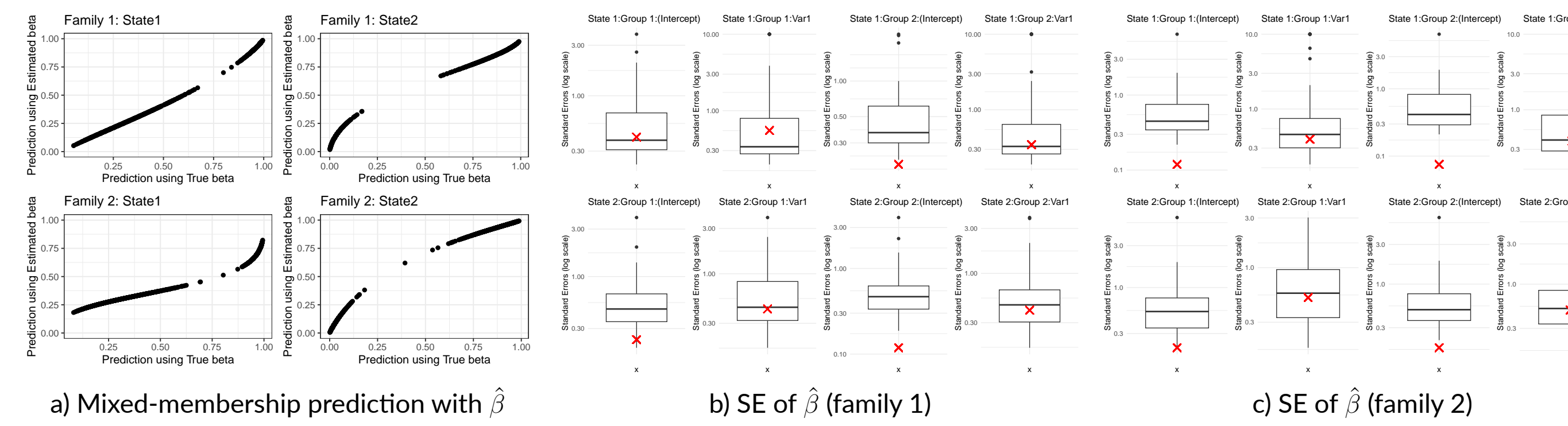
	Easy	Medium	Hard
$\text{logit}^{-1}(\mathbf{B})$	$\begin{bmatrix} 0.90 & 0.01 \\ 0.10 & 0.60 \end{bmatrix}$	$\begin{bmatrix} 0.90 & 0.05 \\ 0.25 & 0.60 \end{bmatrix}$	$\begin{bmatrix} 0.90 & 0.10 \\ 0.40 & 0.60 \end{bmatrix}$
β_1	$\begin{bmatrix} 1.50 & -0.50 \\ 1.25 & -1.50 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.25 \\ 0.75 & -1.00 \end{bmatrix}$	$\begin{bmatrix} 0.60 & -0.05 \\ 0.50 & -0.55 \end{bmatrix}$
β_2	$\begin{bmatrix} -3.00 & 1.00 \\ 7.25 & -4.25 \end{bmatrix}$	$\begin{bmatrix} -0.50 & 0.25 \\ 1.25 & -1.25 \end{bmatrix}$	$\begin{bmatrix} -1.05 & -0.55 \\ 1.55 & -0.25 \end{bmatrix}$

Easy, medium, to hard DGPs: more similar entries in B and more mixed memberships

Medium Case Results:



- a) and b): estimated mixed-membership vectors align with known values;
c): estimated blockmodel match known values (white numbers)



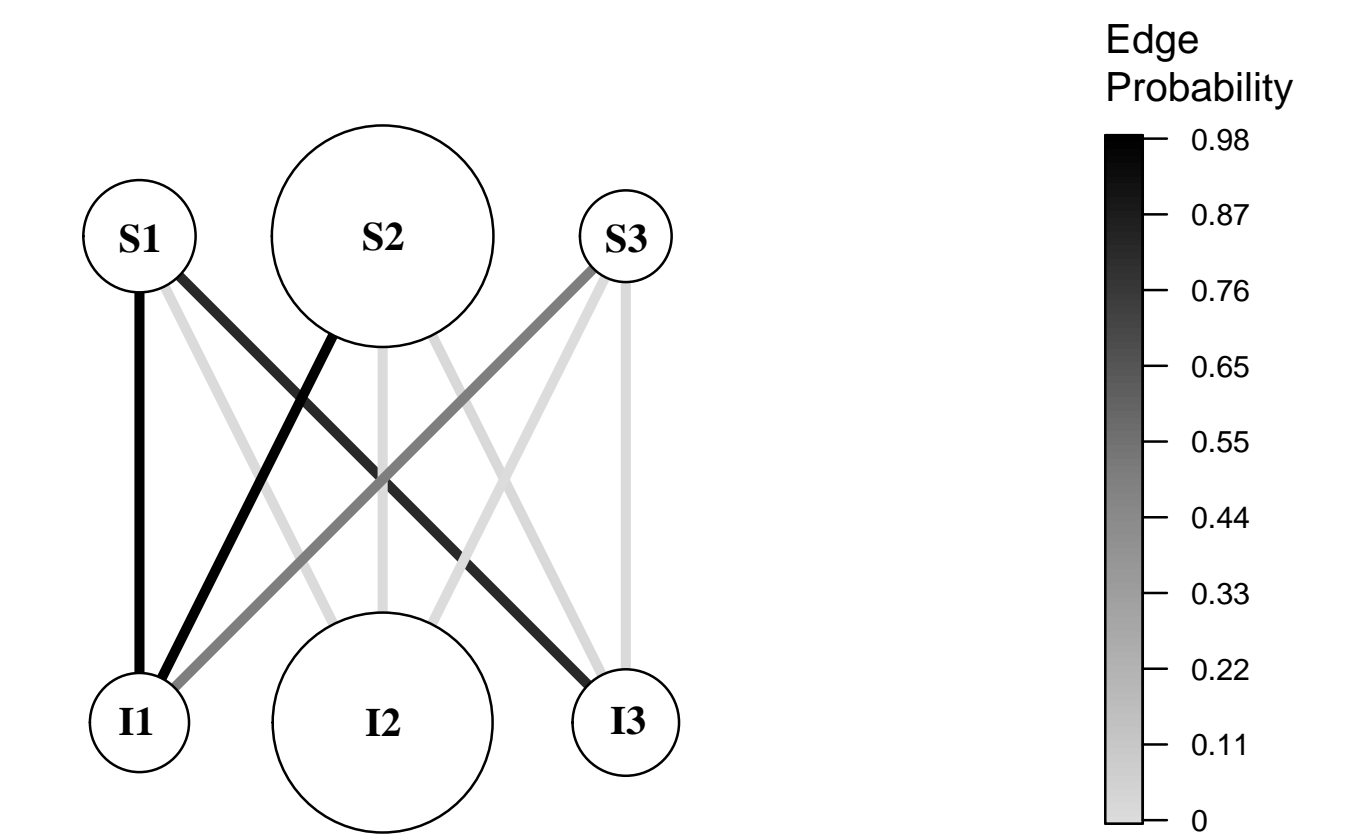
- a): mixed-membership predicted using estimated coefficients match prediction using true coefficients.
b) and c): SEs across 100 simulated networks covers $\text{sd}(\hat{\beta})$ (red crosses) well.

Application: State-IO Network, 1965-2014

- Data:** Yearly state-IGO membership data between 1965-2014, covering 200 countries and regions and 471 IGOs (Pevehouse et al., 2020; Davis & Pratt, 2021).
- Parameters:** Three groups for both the state (S) and IGO (I) families. One latent state.

Group Labels Based on Estimated Block Model

- S1 (Internationalists):** Most likely to instantiate links to a large number of IGOs in I1 and I3.
- S2 (Opportunists):** Most populous group in the state family. Very likely to instantiate links to IGOs in I1, unlikely to interact with I2 and I3.
- S3 (Isolationists):** A small number of states that has a low likelihood to interact with IGOs across groups.
- I1 (Universal IGOs):** Attract a large number of states across groups.
- I2 (Trivial IGOs):** Large number of IGOs that are unlikely to instantiate links with any groups of state.
- I3 (Exclusive IGOs):** A small number of IGOs that only interact with states in S1.



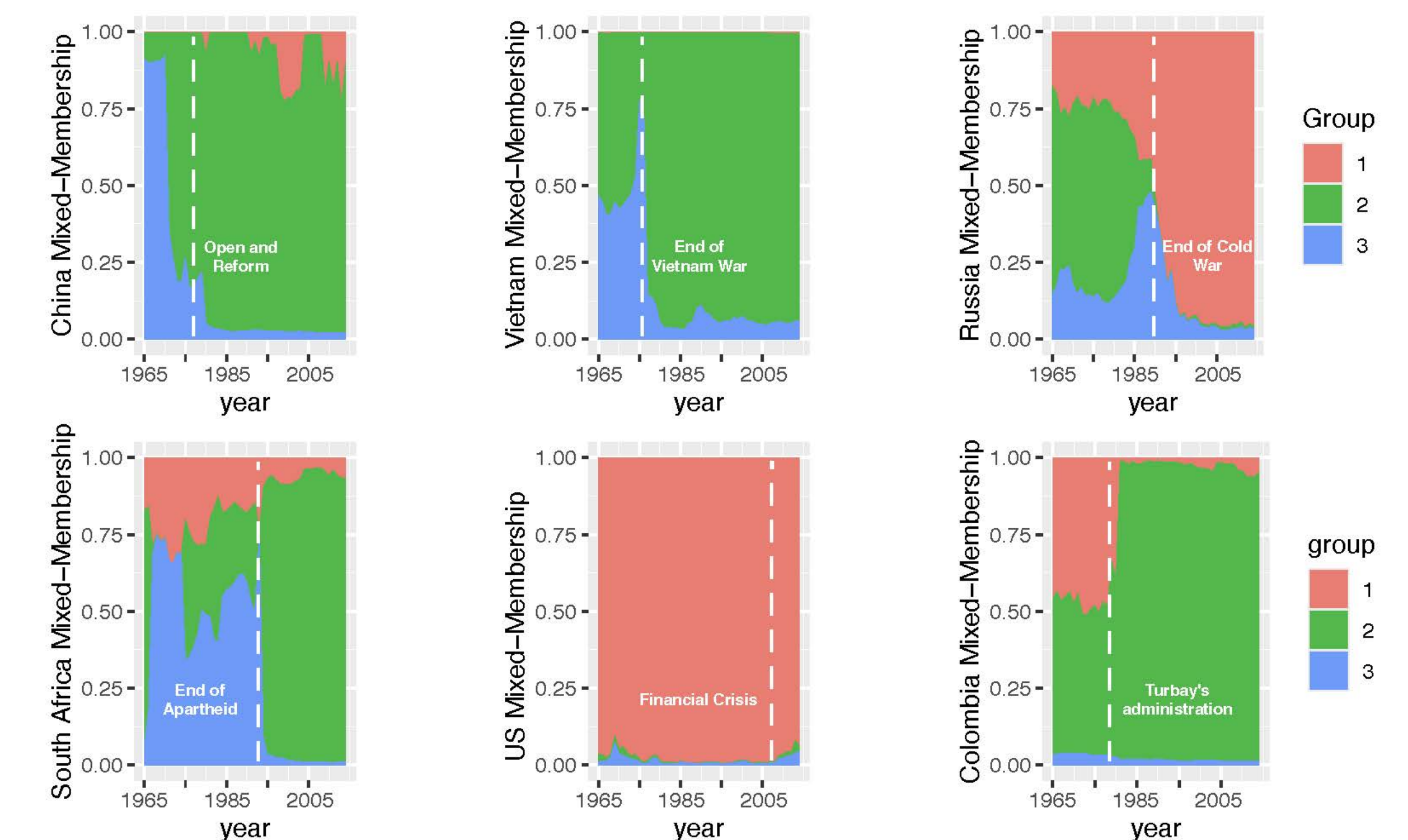
Validating Group Labels

The **covariate effects align with our group labels**. For instance:

- Rich, democratic states that are geo-politically aligned with the US are more likely to have larger share of mixed-membership in S1.
- IGOs whose average members are more US-aligned and cross-regional are more likely to have a larger share of mixed-membership in I3.

Predictor	S1	S2	S3	I1	I2	I3
UN IP	-0.59 (0.29)	-2.26 (0.26)	-0.83 (0.25)			
V-Dem	2.95 (0.63)	3.97 (0.72)	0.68 (0.71)			
GDPpc	0.25 (0.07)	0.02 (0.06)	-0.05 (0.08)			
Europe	1.33 (0.50)	-2.61 (0.47)	0.37 (0.46)			
Ideal point (lagged)				-2.70 (0.29)	-2.73 (0.28)	-0.39 (0.25)
Regional IO				0.72 (0.51)	0.80 (0.57)	-0.05 (0.53)
Mem. Size (lagged)				0.00 (0.01)	-0.05 (0.01)	-0.01 (0.01)
Salient IO				-0.16 (0.57)	-0.12 (0.57)	-0.23 (0.63)
Number of Dyads	1,833,000					

Dynamic changes in mixed-membership align with related political events:



Simulation

Dynamic bipartite networks over 50 periods, 100 nodes per family:

- Latent states (HMM):** Periods 1–25 in state 1; 26–50 in state 2
- Covariates:**
 - 1 monadic covariate per family: $x_{pt}, x_{qt} \sim 0.5 \cdot \mathcal{N}(-1.25, 0.09) + 0.5 \cdot \mathcal{N}(1.25, 0.09)$
 - 1 dyadic covariate: $d_{pqt} = d_{pq,1} + \epsilon_{dt}$, where $\epsilon_{dt} \sim \mathcal{N}(0, 1)$
- Difficulty levels:** *Easy, Medium, Hard* — vary B and β
- Model recovers:**
 - Mixed-membership vectors, group structure, regression parameters
 - Good performance across difficulty levels